SeeingMath
by The Concord Consortium

Technology-Enhanced Teacher Professional Development:

The Seeing Math Project

The Seeing Math Team at
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December 30, 2006

Bringing Mathematical Thinking Into Focus

In Memory of Jim Kaput

Jim was a valued colleague and important contributor to the Seeing Math project. He shared our commitment to creating materials and technology that embody powerful mathematical ideas in dynamic ways to inspire both teachers and students. Jim was a member of the Advisory Board of the Seeing Math Project, and expert commentator on two of the case studies. Seeing Math video cases and software owe a great intellectual debt to Jim's counsel and vision. His commentaries were deep, insightful, and inspiring. One teacher said that initially Jim's comments went right over her head. After the third viewing of the video she realized she had to change the way she thought about and taught equations and equality. She was amazed that she had the capacity to learn something new and surprising about herself and about her teaching. Jim had a way of surprising all of us.

Jim shared a powerful vision: "democratizing access to the mathematics of change." He believed that it was a moral responsibility as well as a social and political necessity to make mathematics accessible to all. At conferences, and in his papers, emails and personal contacts, Jim forged an ongoing virtual symposium, inspiring and connecting ideas and people, often through his puckish sense of humor. He was deeply committed to the toughest problem—how most effectively to teach all students the core ideas of algebra.

This report is based on research supported by a grant from the U.S. Department of Education (R286A000006). All opinions, findings, and recommendations expressed herein are those of the authors and do not necessarily reflect the views of the funding agency. Mention of trade names, commercial products or organizations does not imply endorsement.

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ACKNOWLEDGEMENTS

The Seeing Math project has brought the best resources to bear on the problem of teacher professional development. In order to do this, we had to assemble a superb staff. But we could not have undertaken such an ambitious effort with the help of an outstanding advisory committee, teachers nationwide who tested and critiqued our efforts, institutional partners who continue to offer the courses, consultants and funding from the U.S. Department of Education. So many people contributed time, energy, and heart to this project that it is impossible to even begin to chronicle their contributions. Thanks to all for everything innovative and accurate in the project.

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TERC: Cara DiMattia , Ricardo Nemirovsky.

Specialist Commentators

In the following, the specialist commentators are listed after the course in which they appear.

Elementary Courses

Data Analysis and Probability: Measures of Center—Maggie Martinez Deluca, Bank Street College, Joseph Garafolo, University of Virginia.

Data Analysis and Probability: Using Data to Make Predictions—Lucile Peterson Bank Street College.

Effective Questioning in the Mathematics Classroom—Joan Kenney, Harvard University and Diane Briars, Pittsburgh Public Schools.

Formative Assessment in the Mathematics Classroom—Joan Kenney.

Foundations of Effective Mathematics Teaching—Joan Kenney, Diane Briars, Pittsburgh Public Schools.

Geometry: 2D and 3D Figures— Fadia Harik, University of Massachusetts, Boston.

Geometry: Calculating Area of a Triangle—Judah Schwartz, Tufts University.

Number & Operations: Broken Calculator—Susan Empson, University of Texas.

Number & Operations: Division with Remainders—Marvin Cohen, Bank Street College.

Number & Operations: The Magnitude of Fractions—Judith Collison, Center for Collaborative Education, Boston.

Pre-Algebra: Pan Balance Equations—Jim Kaput, University of Massachusetts, Dartmouth.

Pre-Algebra: Patterns and Functions—Judah Schwartz, Tufts University.

Secondary Courses

Data and Statistics: Aisling Leavy, University of Maryland, College Park.

Linear Equations: Judah Schwartz, Tufts University.

Linear Functions and Linear Transformations: Jim Kaput, University of Massachusetts, Dartmouth.

Proportional Reasoning: Richard Lesh, Indiana University.

Quadratic Equations: Henri Picciotto, Urban School of San Francisco.

Quadratic Functions: Deborah Schifter, Education Development Center.

Quadratic Transformations: Dan Chazan, University of Maryland, College Park.

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Seeing Math Field Test Teachers

There is insufficient room to list all of the dedicated teachers nationwide who participated in the Seeing Math course trials and offered formative feedback.

The U.S. Department of Education

Joseph Caliguro, Program Coordinator, Cheryl Petty Garnette, Director, Technology in Education Programs, Sharon J. Harris-Morgan, Program Manager.

Technology-Enhanced Teacher Professional Development:

The Seeing Math Project

Executive Summary

The Seeing Math project defines the future of teacher professional development by exploiting the transformative synergy of the best research on teaching and learning combined with a rich collection of technologies. Based on a six-year effort to create the very best professional development courses for teachers of mathematics, this report recounts the development and evaluation of some incredible materials that regularly received rave reviews from participants.

Responding to the Crisis

The No Child Left Behind legislation has focused attention to a long-standing crisis in mathematics education: there are insufficient numbers of qualified teachers, particularly in schools serving poor and minority students. The greatest challenge is that effective teaching requires teachers to make fundamental changes in teaching practice, acquire a deeper understanding of content, and become familiar with technology. It is likely that much of the current backlash against the standards is fueled by examples of poor implementation by unprepared teachers. If the nation is to benefit from the increased student mathematics learning promised by the new standards, and for the standards to function as intended, an intensive teacher professional development effort is urgently needed for all mathematics teachers.

This new teacher knowledge is not easily acquired. A sustained program of teacher professional development (TPD) is needed that uses excellent teaching strategies and provides flexibility and a wide range of specific information (Darling-Hammond & Sykes, 1999). The TPD must not only impart good teaching strategies, but should use and model them as well (Corcoran, 1995; Loucks-Horsley & Matsumoto, 1999). Passive teleconferences and videos, lectures, abstract discussions of pedagogy divorced from content, and generic workshops are of little use. Teachers need to be actively engaged in their learning. They need opportunities to practice and reflect together. And they need content adapted to their interests and needs (Schon, 1987; Shulman, 1986). The required TPD must be affordable, accessible, and applicable. For poorer schools, pricey workshops, expensive video libraries, and courses at distant institutions might as well not exist. The time constraints on teachers further require that effective TPD that efficiently addresses national and local content and teaching standards should be available anytime, anywhere.

The Seeing Math project developed 21 online short courses for teachers of mathematics at the upper elementary and middle school levels. Each course features a video case study, using videos of real teachers in real classrooms. The focus of each video is teacher-to-student and student-to-student interactions, including teachers' questioning strategies that elicit student thinking and help make that thinking explicit. Additional video commentaries from math specialists highlight areas of student misconceptions and insights. Activities using interactive software provide course participants with a math challenge, so they explore the same content as students in the videos. Participants are asked to observe carefully their own processes as they work towards a mathematical solution; they share their processes in discussions with colleagues, and are thus exposed to a wider framework for understanding different problem-solving approaches, including those used by their own students.

Seeing Math Elementary

The 12 Seeing Math Elementary courses are based on blended communities of practice, which use both face-to-face meetings and online activities and discussions to reflect on video case studies. Each course is a set of resources that a school-based professional development course facilitator can use to craft a semester-long study of a particular topic. A detailed guide for facilitators provides essential assistance for using the resources.

The courses aim for depth rather than breadth in skill and content areas; they concentrate on concepts from the NCTM standards that are typically difficult to teach or to learn, including fractions, division with remainders, and using data to make predictions. Two courses look at pedagogy, including formative assessment and questioning strategies. An overview course considers foundations of effective math teaching. Teachscape markets these courses and customizes the courses for schools and districts to ensure success.

Seeing Math Secondary

The nine five-week Seeing Math Secondary courses comprise the core units of a first-year algebra curriculum, covering linear and quadratic functions and equations, plus data analysis and proportional reasoning. These online courses employ powerful tools, including videos of students and a national expert in math education, a content-rich math activity and interactive software for solving math challenges, plus threaded discussions guided by trained facilitators. PBS TeacherLine and Teachscape both continue to offer Seeing Math Secondary courses.

Project Assessment

The project undertook large-scale formative and summative assessments. Both qualitative and quantitative studies indicated significant improvements in teacher pedagogy. Teachers also reported important improvements in their knowledge of mathematics. The quantitative studies seemed to confirm this, but the results were mixed. A substantial effort was made to detect student gains a result of their teachers' participation in Seeing Math courses. Unfortunately, student gains were difficult to interpret. Students in one district following the pre-algebra strand had significant gains on total score and both sub-scores from pre- to post-tests. However, this group did not have a control group, so no gain score comparisons could be made across treatment and control groups. Three districts used Seeing Math courses in the Number and Operations strand. In one district, there were no significant student gains for either year of the study, and no differences between treatment and control groups. In a second district, treatment students made significant gains over the second year of the study, and the treatment students had significantly higher gains than did the control students. In the third district, treatment students had significant gains in both years in some of the test scores; however, both mean scores and gains for the control students exceeded those for the treatment students for all scores. Positive results could have been masked by difficulties in administering the assessments that resulted in a smaller N than planned.

What Was Learned

The project succeeded in creating outstanding materials that teachers felt imparted important mathematics and improved their teaching. The many elements of the project—online courses, video case studies, and interactive software—each contributed to the effectiveness of the relatively short but intense courses. We also learned that courses of this design can be marketed; both the nonprofit PBS TeacherLine and the for-profit Teachscape will continue to offer Seeing Math courses as long as there is interest.

PROJECT OVERVIEW

The Vision

The Seeing Math proposal was developed in March 2000 in response to a competitive RFP under the Telecommunications Demonstration Project for Mathematics funded by the U.S. Department of Education. Because considerable funding was available, we wrote a ambitious proposal for a five-year effort to develop the very best possible online courses for teachers of mathematics at all grades K-12 and to make these available to 20,000 teachers in schools serving high-poverty populations. The project design and scale was intended to have a significant impact on the challenge that schools faced of providing a "highly qualified teacher" for every student by the 2006-2007 school year as required by the No Child Left Behind legislation. This provision appeared to be particularly daunting in low-income and minority districts where it is not uncommon to find teachers with almost no teaching or content preparation in mathematics. The Seeing Math project was designed to address this problem by providing teachers of mathematics nationwide with the most effective professional development opportunities informed by the latest research and powered by emerging technologies.

The proposal envisioned a large number of short courses each targeted at a specific standards-based content area. Experience gained in previous courses convinced us that courses lasting 4-6 weeks were best because they allow enough time for serious learning but are not too long for busy teachers. Having a large collection of such courses allows teachers to select courses to match their needs and available time.

The courses would be online, using the scheduled, asynchronous style that we had found so useful in previous projects (Tinker, 2001, p. v), including the Virtual High School also funded by the U.S. Department of Education (Tinker & Berman, 2000; Zucker, Kozma, Yarnall, & Marder, 2003). This design allows participants the freedom to participate whenever they find a short block of time, but within a time window in which other participants are also covering the same materials. This gives all participants an opportunity to share questions and insights in a moderated online discussion. These online discussions are the key to the success of this style of course, and we had discovered how to train the all-important moderators to guide the discussion while maximizing participant discussions and learning (Collison, Elbaum, Haavind, & Tinker, 2000; Elbaum, McIntyre, & Smith, 2002). We planned to augment this style of online course with video case studies of real teachers in real classrooms who are grappling with the same issues and who are using software and other online resources to good advantage. Video case studies were widely accepted in teacher professional development (Copeland & Decker, 1996; Pea, 2001), but had not been used in online courses. The combination of online courses with video case studies seemed to be a potent combination that could address the looming problem of under-prepared mathematics teachers.

Year One Overview (2000-2001)

In June 2000, the Department of Education notified the Concord Consortium of an initial, one-year grant starting in October 2000 that was considerably less than originally requested for the first year of proposed grant. Therefore, the proposed scope of work for the project was significantly curtailed.

With a limited budget and a one-year project, we decided to produce four short courses that featured video case studies. Teachscape was a close partner that agreed to add production value to the videos and market the resulting four courses. After considerable discussion, we decided that the most challenging and important target would be the upper elementary grades, where we would focus on NCTM standards that were identified as difficult to teach or to learn. It is a challenge to complete online courses in one year that include video cases, because of the long times required to plan, shoot, and edit video. The only way this was possible in time was to abandon any hope of including a revision cycle based on field tests that included the videos.

Year Two Overview (2001-2002)

Midway through the 2000-2001 year, the Department of Education informed us that the project would be continued for another year, but that the amount of the grant would be restricted. It became clear that the project might continue for several years, but that the funding would never be assured until four to six months before the start of the next year. This meant that each year had to be planned as though it was the last year of the project, but might also lead to continued funding. This continual uncertainty had to do with uncertainties about available funding, and did not reflect the Department's assessment of the quality of our work, which was consistently praised.

Knowing the project now had a life beyond the first year, the nature of the project changed. We were able to include a testing and revision cycle and we planned the production of three additional case studies. In addition to our production and field-testing, we also began to explore the issue of teacher-created video cases.

Any new materials have to take into account the growing sophistication of the audience. The video case studies that we produced were excellent, but the next step in relevance and impact was clearly videos of participants' own classes. The cost and complexity of obtaining and sharing videos was clearly dropping and we wanted to be able include this as an option in our courses for participants who were more sophisticated. Because the primary barrier to teacher-produced videos was the difficulty of editing the videos and linking in other resources, we joined with an effort at TERC to create VideoPaper Builder, a tool for creating video case studies (Nemirovsky, Lara-Meloy, Earnest, & Ribeiro, 2001).

Year Three Overview (2002-2003)

During the second year, we learned that the project could expect a significant funding increase in its third year. The Telecommunications Demonstration Project for Mathematics Programs had been folded into a new program, Ready to Teach and there was additional funding with this new program. The staff at the Department of Education hoped to use the new funding to increase significantly the budget so that it begins to realize the initial proposal request. Consequently, our third-year budget was larger than the previous year's funding.

As we explored additional activities and directions for the project, we discovered that Louisiana had identified a critical problem with high school algebra teaching. Teachers who were not certified to teach secondary-level mathematics taught almost half the students in public high school algebra courses. The governor declared this a critical issue for the state, and the Louisiana Department of Education mobilized to address this issue. The Seeing Math leadership met with the Louisiana Department of Education staff to assist them in planning an online algebra course that would be taught by a certified math teacher to a classroom of students who were supervised by an uncertified math teacher. This represented an exciting merger of the Virtual High School and Seeing Math. The project could offer a course by a certified teacher while the uncertified teacher participated in instruction and took a parallel online professional development program that would guide the classroom teacher to gain his/her secondary mathematics certification. Unfortunately, there were insufficient funds in year three for this initiative.

The project completed a total of nine online video courses for elementary teachers, adding topics that our collaborators indicated were the highest need. Teachscape modified its technology and marketing strategies, which had a significant impact on Seeing Math production. Meanwhile, increased feedback from the Seeing Math pilot sites helped the production team optimize the course structure and design.

The call for more "scientific research" reached the Ready to Teach program in the spring of the third year, and the Seeing Math project was invited to prepare a rigorous research component. This was an exciting opportunity that was feasible given the increased funding, although coming near the end of a project created logistical challenge that represented a significant change for the project. We designed a research program that could determine whether there were any impacts on student achievement as a result of teacher participation in our online professional development courses. The full plan required two complete academic years and since funding was not assured, it had to include provisions for a one-year assessment that would be less rigorous. It took several months to develop a new research plan and get it approved by the Department of Education. By the time we had something to implement, the school year had started. Although we began recruitment, it was not until January 2004—well into the fourth year—that we were able to implement the plan.

Year Four Overview (2003-2004)

As part of its response to the increased demands of NCLB, the Education Department program staff was able to award a significant increase in Seeing Math funding for the fourth year, more than twice that of the previous year and more than what the project had been awarded in the first three years combined. This offered the opportunity to produce more materials and explore new technologies. But since there was no certainty about continued funding, it was a huge effort to gear up and complete the work in one year.

During the previous year, changes were taking place with a sister project called TeacherLine that was funded by a grant awarded to PBS under the same Telecommunications Demonstration Project for Mathematics program that funded Seeing Math. Both projects were developing online video-based professional development for mathematics teachers, but with different designs and strategies. At the suggestion of the program officer of both projects at the Department of Education, we worked out a collaboration that allowed the projects to work together. The major role of the Seeing Math project was to develop innovative materials and oversee their evaluation, while TeacherLine disseminated them. Since both projects had been developing materials for teachers at the elementary level, but had not yet done anything at the secondary level, we felt that a collaboration focused on secondary teachers would benefit both projects.

The aborted plan developed for Louisiana became the basis for collaboration with PBS TeacherLine in year four. That plan was modified to focus on online support to middle school algebra teachers. The approach was something we termed "close support" and was based on a set of online content modules that would assist algebra teachers by providing online professional development in specific algebra content areas shortly before the teachers were to cover that same content with their students. Ideally, close support would be coordinated with the material that students were going to use in the next week or two. Because it would be impractical to develop variants of courses for different texts and curricula, we had to compromise by developing generic student materials that targeted critical concepts and linkages from arithmetic to algebraic thinking. This is a reasonable strategy because most pre-algebra and ninth-grade algebra courses cover the almost the same content.

With an increased budget and scope of work, we needed to hire new staff, and requisite project start-up activities took front stage. Our now-expanded project expanded to include secondary level content, which necessitated merging new staff with an established team. Separate production teams were created for the elementary and secondary-level courses, with a small number of personnel dividing time between the two. We continued to develop new courses for the elementary level, and at the same time, began producing a set of "close support" modules for secondary math teachers.

Because the Seeing Math content specialists believed that secondary math is more targeted and conceptual in its approach than elementary math, we decided to build Java-based interactive software applications that would target key ideas in secondary mathematics that are critical to development of algebraic thinking and particularly difficult to teach. These so-called "interactives" would also provide student content needed for the "close support" strategy. Having teachers learn to use the software in preparation for teaching it would provide a powerful motivation for the teacher to learn the underlying content. Thus the software applications would serve both teachers and students.

We focused on late middle school and early secondary where students make the transition from arithmetical thinking to algebraic thought. The framing idea for the Seeing Math secondary courses was the centrality of the concept of function. Few algebra texts for this level deal with the idea of function, or if they do, they offer a formalized, set theoretic exposition that is divorced from real-world application.

During this year, the Seeing Math project took over the publication and distribution of the Balanced Assessment materials developed by Judah Schwartz of Harvard University (Schwartz & Kenney, 2002). These materials consisted of over 300 innovative assessment tasks for K-12 teachers developed over the course of a ten-year project. These materials were used to further enrich the online courses and provoke fruitful online discussions about how to assess students and the theoretical ideas behind balanced assessment.

In response to market needs, Teachscape, our elementary-level partner, changed its delivery model. They found that their clients wanted more flexibility in the delivery of teacher professional development. Our pilot program users expressed a similar desire. Teachscape was planning to enable their delivery platform to more easily enable that flexibility and began to design for the learning object approach to online learning.

Teachscape marketing staff reported a problem when presenting materials to potential clients, as Seeing Math had few course titles compared to other subject areas. When clients examined the courses, they were sometimes intimidated by the level of understanding of the mathematics presented. The Seeing Math team responded with a more flexible, more accessible design for the courses. The design followed suggestions by Deborah Ball and her colleagues that there is a kind of mathematical knowledge specific to teaching mathematics. The Seeing Math design recognized and communicated the idea that central to effective teaching is a mastery of a specialized mathematical knowledge needed by teachers to teach mathematics well. This specialized knowledge was more challenging than the content teachers had come to expect from traditional courses in mathematics offered for professional development. We successfully made the content less intimidating without abandoning the idea that content was important. Teachers were able to make much needed connections between their own mathematical knowledge and the knowledge they needed students to master.

Year Five Overview (2004-2005)

In the final year of funding the Seeing Math project completed 12 short courses for elementary teachers and nine for secondary mathematics teachers. The additional courses for elementary teachers came in response to Teachscape's identification of a need for a new course design, which we called "orthogonal cuts." Courses based on "orthogonal cuts" recognized the importance of insights from the Horizon Research report "Looking Inside the K-12 Mathematics and Science Classroom" (Weiss, Pasley, Smith, Banilover, & Heck, 2003) and focused on the development of high-quality questioning, quality formative assessment, support of mathematical rigor, support for student mathematical thinking, and support for student ownership of ideas. Footage from existing courses, and previously unused material was repurposed to develop two new courses that would meet teacher's needs. A third new short course was developed to serve as an introduction to mathematics instruction and professional development at the elementary level. This overview course was also designed in response to difficulties that Teachscape had identified in marketing Seeing Math materials.

At the secondary level, the project produced a final set of courses and refined the first group of courses. The project also explored making both the course content and the project website more accessible. A new Seeing Math website was created, which garnered a surprising level of publicity. The collaboration with PBS to market the courses continued and all the secondary courses were transferred to TeacherLine.

The strand of development designed to provide a tool for teachers to develop their own video cases culminated when VideoPaper Builder version 3 was released. The changes made incremental editing possible, greatly simplified the production of video cases by teachers for use as a tool for teacher professional development. While not incorporated into Seeing Math courses, there is a large community of educators at the post-secondary level using the software for a number of purposes, including portfolio assessment, reporting, and as an instructional tool.

Year Six Overview (Continuation Funding, 2005-2006)

The funding for Seeing Math terminated at the end of September 2005, but a no-cost extension was obtained for a final year of activity. No new materials were developed during this year, but a variety of research and dissemination efforts were undertaken. The major activity was to complete the research studies. The design of these studies required two years and had only begun in earnest in year five, so a sixth year was essential. The field tests were completed and the results analyzed and reported by our external research partners.

In addition, we formalized agreements with both Teachscape and TeacherLine for the continued distribution of the secondary courses and transferred to each the complete set of files. Finally, some technical enhancements were made in the interactives to make them easier to use with students. A number of efforts were made during the year to increase general awareness of the Seeing Math resources available to educators: the free interactives and sample courses on The Concord Consortium site and the full moderated courses available at Teachscape and TeacherLine. This effort earned an "A+" from Education World.[1]

Course Design and Content

Supported by a media-rich context and interactions with significant mathematical ideas, Seeing Math participants work together to generate new ideas, build new connections, and extend their understanding of both math content and pedagogy. The ability to learn from each other greatly increases the likelihood that large-scale implementation of professional development in secondary mathematics is feasible. This section describes the course design that fosters this rich learning experience.

Common Features of all Courses

All 21 Seeing Math courses share the following characteristics:

1. Authentic video depicting real classroom situations

   Stimulates observation, analysis and reflection, encouraging teachers to refine their own practices and content knowledge.

   Provides valuable insight into student thinking about mathematics.

2. Expert commentary by nationally recognized specialists

   Highlights relevant standards, learning theory and instructional practices.

   Offers ideas about how teachers can improve their professional practice.

3. A wealth of online resources to complement each case study

   Connects learning experience to classroom realities with sample lesson plans, examples of student work, and references to corresponding state standards.

4. A research-based curriculum developed by experts to address national and state standards

   Equips teachers with content mastery to effectively prepare students for high-stakes, standardized testing.

   Draws from years of research on the use of technology-based learning and math education.

5. Innovative, interactive software that illustrates and reinforces mathematical concepts

   Accelerates learning and enhances comprehension of difficult concepts.

   Equips teachers with software that makes math come to life in the classroom.

6. Moderated online discussions

   Encourages thoughtful, probing contributions and guides dialog to key learning points.

   Builds community through collaboration and shared learning.

The elementary courses are designed for site-based professional development where several teachers can gather regularly for face-to-face meetings. This calls for a blend of web-based and face-to-face

[1] See http://www.education-world.com/awards/2005/r1005-19.shtml

learning experiences. On the other hand, secondary teachers are less likely to be able to meet and are more focused on content. For them a 100% online course is more effective and convenient.

The 21 courses are described in the Appendix and on the Seeing Math website.[2]

Structure of the Seeing Math courses

The metaphor guiding course construction was the experience of a visitor to the exhibit rooms of an excellent science museum. The Seeing Math experience, like an exhibit, is conceptually and intellectually engaging. The design shifts the online museum guide from instructor or potential content provider to that of a moderator who monitors each participant's progress through the exhibit, encourages participation, troubleshoots technical difficulties, and, through private feedback, provides guidance on topics that may need more attention.

Each course is comprised of five week-long segments. The first week introduces the course and the platform. After a community-building activity, participants engage in problem solving through a "Diving In" activity, which features significant mathematical content embedded in an interesting problem. An interactive applet displays unique representations of the problem and new ways to think about solutions. Participants can share their solution paths with each other in an online threaded discussion area that supports text as well as pictures and snapshots from the applet. In week three participants view two or more short videos of students tackling the same problem. The students' impasses and false starts often mirror the participants' own struggles. In weeks three and four, a math specialist comments on the students' efforts and highlights important content issues, potential unresolved areas of confusion, and links to current research. In the threaded dialog, participants integrate multiple solution paths and complexities revealed by the students and the expert commentary, and discuss insights from the linked multiple representations displayed by the applet. As a summative experience, participants design or adapt course activities for their own curriculum.

The Role of the Moderator

The Seeing Math Secondary course design follows the Concord e-Learning Model,[3] which describes nine key characteristics of quality online courses. The role of the moderator in the Seeing Math model diverges significantly from that detailed by Feger and Zibit (2005) who build on previous researchers and detail three areas of "teaching presence" or focus for the online moderator: 1) instructional design and organization, 2) facilitating discourse, and 3) direct instruction. The "co-construction" moderators they describe employ a facilitator-mediated discussion design that includes supporting lesson study processes, supplying teacher resources and context, and coaching and addressing cross-grade issues. This model falls short of the goals and achievements of the Seeing Math design in notable ways. The co-construction model proceeds dependent on significant levels of moderator analysis, input, and intervention that potentially hinders independent thought and development.

The Seeing Math design recognizes that high quality expertise is needed to foster deep engagement in the content. The Seeing Math architecture offers alternative, scalable sources for this important course element by expanding on key ideas, conceptual conflicts, ambiguities, and unresolved issues

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[2] http://seeingmath.concord.org/

[3] http://www.concord.org/courses/cc_e-learning_model.html

within the text surround, video commentary, Diving In activity, and student video. The expert commentator in the videos takes on many of the characteristics of a co-participant in the group, and also serves to assist the moderator by focusing the discussions. Access to this valuable window on content depth shifts the moderator out of the center of discourse with the participants. The applets, using linked multiple representations, also provide a powerful way for teachers to approach old ideas in new ways. The Seeing Math discussion boards abound with teachers' discoveries within this media-rich environment.

Online courses that use discussion boards are often exhausting for course moderators and, therefore, not easily scaled, because the moderators have to carry the conversation, often posting as many messages as all the participants combined. Seeing Math is completely the opposite. Statistical analysis of the dialog in three Seeing Math course sections offered in the spring of 2005 revealed an unusual moderator profile. The participant-to-moderator response ratios were quite high, ranging from 19:1 to over 100:1. A single, targeted comment by the moderator often generated considerable discussion among participants. The software platform required that the moderator place the initial post in each thread. This post was scripted by the course authors and provided in the facilitator's guide. It highlighted the main themes for discussion and the important conceptual tensions evident in the student and expert commentary. In a study of three courses, 1,529 posts were made by a total of 53 participants. The moderators posted 39 scripted messages and 18 unscripted messages for an average participant to moderator postings ratio of 27:1. By comparative word count, moderators occupied only between 1% and 2% of the public discussion areas. Over 50% of the postings occurred in threads containing 11 or more posts, and the average word count of a participant entry was 108, indicating considerable participant activity in the online discussions. This high level of participation is due to the rich materials and course structure that gives participants many experiences that they want to share, without requiring the moderator to draw them out.

Seeing Math moderators exert considerable formative influence on the course outside the discussion area as well. They spend significant time resolving technical issues, such as difficulty installing video players or the proper version of Java to use with the applets. They also give weekly feedback to participants in their private discussion areas. Moderator feedback includes highlighting participants' contributions that received significant attention from other participants and suggesting where more effort to communicate or articulate ideas could benefit others in the course. Moderators received seven weeks of training based on the ideas in The Online Teaching Guide and Facilitating Online Learning, which familiarized moderators with "voice" and "tone" and a set of critical thinking strategies. Technical training was also included.

Developing the Course Structure

For the first two courses, which were in elementary math, Seeing Math used Teachscape's delivery platform and its course template as starting points for content creation. The core of the case study featured in each course captured a real-life video narrative, presented the problems each teacher faced, and conveyed the solutions that grew out of imperfect situations. The storyline integrated two essential elements – math content aligned with NCTM standards and classroom pedagogy. Watching and analyzing the way teachers made decisions about their teaching led participants to make better analyses and decisions about their own teaching. The template also included reflective activities that fostered online discussion and/or face-to-face dialogs, commentary by the teacher highlighted in the video, a math specialist's commentary, state and national standards alignment, software interactives, readings, online resources, and final project rubrics for those participants who took the courses for graduate credit.

While this structure provided a powerful and robust framework for the courses, we realized that we were missing a crucial element – the course participants' active engagement in the math content. This led to the creation of the "Diving In" Activity, a problem that simultaneously introduced the math topic and modeled our approach to learning/teaching the math content. It also prepared teachers for understanding a similar problem that students in the videos encountered. Diving In was the mathematical heart of the course. In these and all subsequent courses, the Diving In activity took the form of word problems, animations to play and examine, data collection activities, or interactive computer tools, and its two-fold purpose was always the same: 1) to make the teachers wrestle with the same problem the students wrestled with and use it as a window into their own thinking, and 2) to let the teachers understand the problem from the students' point of view, preparing them to observe the students in the videos who encounter the same problems.

A second element that evolved during the development of early courses was the specialist commentary. In the beginning, we asked an expert in the math education field to point out "good things" the case study teacher did with her students, explaining why the teacher showed exemplary professional practice. During the development and production of our third course, however, while screening the classroom video, the specialist did more than simply observe the positive things demonstrated by the teacher. He began to think out loud, analyzing the powerful math principles underlying the problem the students in the video were solving. He also began to describe new ways to pose the problem that would illuminate the concepts on which the existing solutions were based. This was a seminal moment. Our next step was to incorporate his mathematical insight into the video segments. This helped course participants see beyond a single case study experience and understand other mathematical approaches to the same problem. This became our standard – video commentary that took course participants' understanding to new heights, commentary that carried the power of the expert's voice with mathematical explanations augmented by high–quality graphic animations. In the course dialog the targeted commentaries of the specialist functioned as springboards for teacher reflections and pushed dialog much deeper.

In the second year, as we took stock of our growing library of course materials, met with district facilitators from pilot implementations, consulted with our advisory board, and reviewed formative feedback, we saw the value of computer-based applications for teachers in the Seeing Math courses. In response, we developed a course based on student use of interactive software. We provided teachers with a new software tool, the Broken Calculator, which they could use with their own students and could be used in the Seeing Math course to model effective teaching strategies for teaching number and operations. We also included the software in a video case with a teacher who taught a heterogeneous class of students with a wide range of math abilities and attention spans to show how the technology can be used to motivate a variety of students. The positive response to course integration of a software tool and video of students using that tool set the stage for developing Seeing Math Secondary materials.

In the early design stages of the elementary courses, we worked with the Teachscape marketing group to understand how the courses would be used. Teachscape's marketing model was to sell a library of courses to schools or districts. Curriculum department chairs or others responsible for district professional development would take charge of implementation, customizing the materials to their specific needs. Therefore, Teachscape wrote a user guide that outlined the technical needs for accessing materials from their delivery platform and suggestions for running face-to-face workshops within school buildings. With that starting point, the Seeing Math team was asked to write "discussion prompts," similar to the prompts that Teachscape curriculum developers had written for the science and literacy courses in their library. We soon found, after talking with local facilitators and district professional development teams, that the Seeing Math content was complex enough to demand more than simple discussion prompts. Users and facilitators needed more guidance than provided by suggestions for discussion seeds. They needed to know how to facilitate a discussion in order to draw out salient pedagogical points as well as to lead teachers to understand the deeper mathematical principles involved in the case studies. Over the third year, we developed a template for all our courses, and created separate guides for the course participant and the facilitator.

When first considering how to guide online and face-to-face discussions based on course content and following good pedagogical practices, we deliberated over how to appropriately divide online and offline interactions for optimal teaching and learning experiences. We also thought carefully about how long each of the five to six "sessions" within each course should take to glean the maximum value from the materials. The guides reflect that careful thought and subsequent design. When the pilot schools used the guides, we learned that it really did not matter how many sessions the materials were divided into. Nor did it matter whether discussions took place face-to-face or online. What mattered was the quality and the depth of the discussions – online or offline – and whether the material was covered, reflected on, and shared with other colleagues.

Consequently, revisions focused on de-emphasizing the scheduling elements and easing specifications for online or offline interactions. What remained core to the guides were the key mathematical and pedagogical issues that needed consideration such as, "What is the difference between formative and summative assessment?" "How can you move a student from additive to multiplicative thinking?" "What constitutes effective questioning strategies?" "How many ways can a remainder be interpreted depending on its real-world context?" In subsequent years we heard from our users that they appreciated the flexibility of the materials, which allowed them to customize the courses to meet the unique needs of their respective schools and districts.

Toward the end of the third project year, as we completed the core set of Seeing Math Elementary courses, we started to plan the Seeing Math Secondary courses, focusing on topics typically found in Algebra I courses. We analyzed what early users and independent evaluation of elementary courses told us about which aspects were effective and led to positive change in teacher practice. We also analyzed which medium or technology encouraged the most thoughtful inquiry and exploration, or the clearest communication, or the most considered reflection. Because elementary teachers are generalists whereas there are almost always math specialists in middle schools, the nature of professional development is quite different in these grades. These considerations resulted in the following three-part instructional design.

Focus on math content. Users started with readings and hands-on math assignments. They completed a series of math challenges that embodied key concepts, and they were encouraged to use varied approaches and representations. We learned from the Diving In activity in Seeing Math Elementary how valuable it is for teachers to wrestle with the same ideas their students do. We also learned from the Broken Calculator course the value of interactive software that not only provides multiple representations, but also provides a tool for multiple approaches to problem solving. Therefore, we developed interactive software tools to use with each Seeing Math Secondary course, giving teachers a way to experience math concepts in a new light, in addition to giving teachers new resources to use with their own students.

Observing student thinking. In the next portion of the course, teachers watched videos, listened to specialist commentary, and completed readings in order to attend to student thinking and to deepen their understanding of student approaches. We learned from our elementary courses that videos of students at work paired with specialist commentary encouraged participants to replay videos and review key classroom interactions, enabling the teachers to perceive and reflect on things they might have missed during the first or even second viewing. Leveraging the power of the case-based teaching and learning that we used in the elementary materials, we redirected the user's attention away from what the videotaped teacher was doing to listening carefully to what the students were saying. In addition, the videos often showed students using the course software solving similar problems that course participants themselves had solved previously. This showed them new approaches developed by students, students struggling with concepts that teachers did not realize could pose difficulties, and increased teachers' empathy as they observed students working to overcome the same hurdles they had.

Applying course learning to classroom practice. The third part of the instructional design enabled the teachers to take the insights about their own math content knowledge from the first part, couple it with insights into student thinking from the second part, and then examine how their particular school curriculum approached the same content. This process culminated in course assignments in which users synthesized their learning to create a plan that put the course ideas into their actual classroom practice. In this section we felt it important to let the teachers choose the technology most appropriate for them to create and share their plans, whether it be a set of word problems, a plan to use the software interactives with students, or a hands-on activity. The teaching and learning goals drove the technology, and not the reverse.

After meeting with the National Center for Accessible Media group at PBS in Boston, we redeveloped our courses to meet Section 508 standards. These features included captioning and transcripts for video and audio content, alternative text representations for images, and other changes. NCAM worked with us to make the case studies presented on our site accessible for verified educators.

Teachers Speak

The question of the effectiveness of the resulting Seeing Math course designs will be addressed quantitatively in a later section on evaluation. However, the voices of participating teachers convinced us that we were on the right track long before the formal evaluations were complete. Below is a sample of the comments we received. They illustrate how the course design elements combined to produce a powerful impact on teachers in both their teaching style and the content of instruction.

Changing Teaching Style

Many teachers report that the experience of taking a Seeing Math course would improve how they teach.

> *I am a better mathematics teacher because of the two modules we did.*
>
> *The most important thing I have learned from this course is how much I have to change in my teaching style. As I have stated before, I am an old fashioned teacher. Skill & Drill. That has to change. My students need to be given more opportunities to explore, discover, and think like never before. This will take a large adjustment on my part, but I am determined to succeed. The problems and ideas I have gathered will be a tremendous help in the future.*
>
> *The course presented some basic core concepts in a very hands-on, productive way. I love the stuff.*

Learning from Peers

The online discussions create communities that were central to the learning experience. Of course, this does not happen just by signing up a group of teachers. The following quotes from three teachers are enthusiastic because the course provided the structure and content that enabled rich discussions.

> *Here's what I valued most about this course: The excellent classmates! I gained so much from seeing others' perspectives on our given tasks and issues! I really loved the interplay of our postings and responses, and the fact that all of us had all the time we needed to read each others' views, mull over our own views, and respond in our own timeframe.... in a regular class, if you don't have your thoughts all formulated at the right time in the discussion, you can't just rewind the tape and ask the question or add the ideas later! It was great, too, for the shyer of us to have the anonymity of a posting rather than a face-to-face interaction, and the knowledge that even the most vociferous critics couldn't sneer in our faces, so we could all share our insecurities, questions, and even occasional criticisms with increasing comfort. It makes me wonder again and again, how I can make my OWN classes, with my middle school students more closely fit this model.... What can I change about my more conventional class (we all meet at the same place, face to face, at the same time...) to bring the vibrancy and individualized learning opportunities I had the pleasure to experience in this class?*
>
> *Whew! Well I started this course not knowing much about on-line courses. I have found it to be a very neat experience. I have really enjoyed the conversations exchanged and perspective each of you brings to the table. I thought the reflection questions place strategically throughout the activity was one of those things that just made such sense. I will be using that in the classroom when completing labs. Selecting appropriate teams came back out to the forefront. I think my students will benefit from a more thoughtful placement to include verbal and non-verbal learners. I just want to say that I have learned most from all the discussions I have enjoyed getting to know each of you.*
>
> *Although we all work with other math teachers, it seems there is never enough or the "right" time to bounce ideas off of one another as we have here. Everyone in this course has become an important part of my professional community and I truly appreciate each of you. I know how important time is to all of us so I want to thank everyone for the time they have put into making this course such a wonderful learning experience. One of our advertised goals for graduates in my school district is that we produce a community of "life-long learners." My participation in this course has allowed me to model this behavior for my students.*

Understanding Students

Putting teachers in the role of learners and then asking them to reflect on what it was like for them is a powerful strategy. The interactive software presented a new environment to explore where participants could be the learner. This experience was frequently mentioned as eye-opening, even for experienced teachers, as the following testimony from different teachers attests.

As I reflected on (struggled with) my own solutions to the broken calculator, I was aware of my own levels of thought and realized the value of such a learning activity in my classroom. The notion that there are many viable strategies that cause deeper thoughts about the math takes math instruction away from rote algorithms and toward critical thinking and application.

The activities are great... they really make you re-examine the way you teach the topics in your own classes. Things like the Qualitative Grapher interactive have exciting implications for use in the classroom. The opportunity to interact with so many colleagues on a regular basis is invaluable.

The course really helped me to be more aware of potential areas of student misunderstanding, and helped my teaching as I was taking the course.

Walk a mile in my shoes...." This class gave me the opportunity to "walk a mile in my students' shoes." I was reminded of what it is like to struggle through a concept and not always have "the right answer." I was reminded of the wonderful feeling of success at those "aha!" moments. I was also reminded that sometimes your best teachers are your peers. I tend to teach fairly traditionally because that is how I was taught. One of my goals as a new teacher is to try something new each trimester - a new activity, a new technology tool, etc. This class provided ample opportunities and ideas to do so. Thanks to all of my classmates whose wealth of knowledge and experience opened my eyes to those new opportunities and ideas. Thanks for listening, and thanks for sharing. Best wishes to you all.

What a great experience this has been. I've learned a lot and expanded my thinking in algebraic topics, but this course has also allowed me a different perspective. As a teacher, I think we sometimes need to be to reminded what it's like to be the student again. The frustrations of problem solving, learning new concepts, and completing assignments are often forgotten by the teacher. It was good to have a reminder of what it's like to be in the students' shoes again.

I have learned so much in this course that I can hardly wait to share it all in the classroom. I know my understanding will deepen and become richer as I teach my students the concepts I've learned. My own struggles in the learning process here have reminded me of what challenges many of my students face every week. I hope I have learned more empathy and understanding, and I'm excited to put into effect the strategies I've learned in this course. I owe so much to you, my "math colleagues". Your conversations have opened my eyes many times, and guided me through my own difficulties often.

Every time that I take one of these courses, most of it seems new until I come to the end, and then realize that I knew much of it at the beginning. Now, I know another way to approach the same material and have learned some more. I feel that having "math phobia" makes one feel like they know nothing. I can imagine how my students feel. I am just starting to shed some "math phobia" layers, but when it comes to something new, I still have to take a breath and tell myself that I can do it. After learning something new, it makes me excited.

I find myself expecting my students to see concepts easily, like I do. Well, I am 38 and not 11, and have been doing this a lot longer than they have. Setting down and working on these problems like a student really helped me to better understand where they are coming from when they learn new things. I am very comfortable with the things I teach and the way I teach them. Seeing and attempting to understand the way other people go about solving problems has got to be very similar to what my students are feeling when they watch me process my understanding. Thank you all for allowing me to watch you the way my students watch me.

Encouraging Student Thinking.

The lesson many participants took away from their experience as learners in the Seeing Math courses was to foster for students the kind of thinking and reflection that they found so valuable for themselves in the course.

> *I have heard over and over that a reflective teacher is an effective teacher. Well, this course demanded deep reflections and got it. Many times I had to dig deep to come up with my responses to the assignments - deep mentally and physically into the night! But I think the reward is worth it: I am coming off with a bag full of great teaching ideas scraped together from great teachers and specialist, online interactives tools, tons of ready-to-go activities and a deepened understanding of algebra!*

> *This course helped me learn something about myself. When I approach math, I have a tendency NOT to "take the risk." I am afraid I might make a mistake. When I began this course, I was afraid to say certain things because I thought others might think I was nuts. I soon learned that some of the misconceptions, thoughts and ideas I had were just like those of my fellow colleagues. So, how does that reflect in my classroom? I might think I am letting the kids "explore" math, but what can I do to encourage it more? I have given this a lot of thought and hope to use what I have learned in this course to encourage "math talk" and the "inner learner" in all my students. The very concept that yields real gold is the "function" and how it relates to the linear equation.*

> *"I have realized many times during the past four weeks that the best learning experiences that have occurred in my classroom the past 30 years...." I could complete Dan's thought with "...never happened because I didn't give myself the opportunity to listen to my students, didn't try hard enough to see what they were seeing, and didn't help them find solutions to the problems THEY saw." That's the "Oops!" The "Aha!" is what I've learned in the last few weeks about listening to my students and giving them the opportunities to learn with creative questioning.*

Making Math Real

The Seeing Math courses attempt to balance content and pedagogy and we see in participant comments growth in both areas. One theme that participants appreciated was moving math beyond abstractions and connecting it to the real world.

> *This course has made me connect to my curriculum. At the rate we teach concepts, I often leave out the most essential piece to student learning - real-world applications. Before this course I would have just shown students HOW to find slope, intercepts, intersection points, and domain and range as well as many other things. Now, I will show my students that this stuff really can be applied to their lives. I have enjoyed this course and will try to take the one on quadratics before I teach the concept at the end of the school year.*

The Central Role of Functions

Another content theme that resonated with teachers was emphasizing the central role of functions.

> *Where have functions been in Victoria, Texas? Chapter 13 that's where! BUT now and forever more they will be at the beginning, middle and end included everywhere and all year long. This was also the first time I have taken an on-line class – it was great.*

> *Functions have been completely absent I my curriculum. They are not touched on until the 12th chapter, which I have never gotten to during the school year (not enough time) I have finally understood what a function is and am already adding the term and definition to my classroom work. My students will not go on to HS without hearing the word, and they will have at least the beginnings of an understanding of it.*

> *I never really taught functions and linear graphing in conjunction. I taught functions and how to find the equations, but left the graphing as a different concept. Now I will help my students see this connection and teach*

them together! I have done a really base job of teaching linear graphing. I know I need to be more thorough and draw more connections this time around. I love having more options to use technology in the classroom. In a lot of ways I think these are easier to use than graphing calculators.

While I've worked with functions extensively teaching the upper levels - particularly pre-calc and calculus - I never really thought about how powerful they are when used right from the beginning. While it is important to understand how to manipulate and solve linear equations, once we move into graphs and contextual problem solving the use of functions become apparent and will make life so much easier for students to understand. Heck, it made it easier for me to understand, and I already know this stuff. I can no longer use the text-book's notion that functions are an afterthought, relegated to the back of the text, nor can I ignore the curriculum in which functions are not taught until Algebra 2. I need to introduce functions as soon as that first y=mx+b equation shows up and use functions as a springboard for everything else we do

Before this class I did not feel very powerful where functions were concerned and I was even reluctant to commit to taking this class. However, you my peers have given me power. Through your knowledge, comments on the discussion board, struggles and accomplishments, you have helped me become a better teacher. We all get in a rut and comfort zone when teaching so taking this class with such a talented group of people has been a wonderful experience. I have learned so much with you and from you and am excited to proceed forward with my teaching of functions. To all you Texas teachers, you are great. I wasn't sure about on line courses, as this was my first but will not hesitate to take another one.

TECHNOLOGY DEVELOPMENT

The project began with a content development effort that included little technology. However, over the course of six years, technology became a major effort. The Seeing Math Technology group researched course management platforms that could best deliver video, interactive software activities, and text files, as well as provide an environment to exchange thoughtful, facilitated discussions and completed course assignments. The research informed the dissemination partner for Seeing Math Secondary, PBS TeacherLine, in its selection of a new course delivery platform. This section also covers the design and development of the Java-based interactive software that secondary teachers and their students used in studying pre-algebra, algebra and statistics. A parallel development with VideoPaper Builder software is described.

Interactives

Judah Schwartz (1999) challenged mathematics educators by asking “Can technology help us make the mathematics curriculum intellectually stimulating and socially responsible?” Schwartz envisioned a major paradigm shift in the teaching and learning of mathematics in which web-based technology would add representational power and thus new dimensions to visualizing, as well as capacity to communicate mathematically with others. The Seeing Math project, was greatly inspired by that vision and planned from the beginning to incorporate interactive software that would help both teachers and their students master important concepts. The Seeing Math software design represents functions, data sets, and proportional relationships in a way that is not possible on current handheld devices, like the graphing calculator.

Because we did not want the cost of software to inhibit teachers from using it freely in with their students and their parents and friends, we wanted to use only open source software. We found that no open source software was available that focused exactly on the content of the courses, so we created our own. As with all our software, we developed it in Java because this language offers good performance across all the operating systems found in classrooms.

Researchers have identified two qualities of applets (small Java objects that are delivered by the Web) that make them effective cognitive tools to support growth of understanding: interaction and interactivity (Nanda, Liang, & Sedig, 2005). Interaction characterizes the "conversation" between the tool and the learner. Seeing Math interactives support the learner/applet conversation in many unique and highly effective ways. In some, users move a slide bar below the x-axis to draw the graph. The design emphasizes the function as the link between the independent and dependent variables. Users can shift readily from one algebraic form of a function to another equivalent form by clicking a tab. The graph of a function and its symbolic representation are dynamically linked, so changes in one affect changes in the other in real time. For instance, vertical or horizontal shifts (transformations) change symbolic representations. The Quadratic Transformer enables users to experiment directly with f(x) notation. Questions like "How is a(f(x)) different from f(ax) or from f(x-a)?" become the focus of individual and group inquiry. Immediate feedback through linked representations draws users deeper into exploration of mathematical content.

Interactivity refers to the feel, form, properties, and quality of the interaction with the tool. Seeing Math applets permit online sharing of screen shots, which supports online dialog. Because they explicitly and dynamically link graphical representation and symbolic and numerical forms, the tools abound with "what if" possibilities to explore mathematics. For example, the Piecewise Linear Grapher permits students to define linear relationships that are not functions. If a user has created a graph with multiple y values for some values of x, a warning box indicates the double y-value assignment for that region, but does not suggest how to fix the problem. The animation of the function simply disappears for the region of the domain for which the function is improperly defined. The design turns an annoyance (the disappearing animation) into a learning opportunity. The student asks herself, "What changes in the domain will fix the problem?"

To simplify their use with students, all the interactives are available separately from the course at no cost.[4] A brief description of the eight interactives can be found in the Appendix.

The eight Seeing Math algebra interactives support an approach to algebra using the function concept as a central theme. Traditional approaches offer sets of exercises detailing proper manipulation of symbols and equation solving. Teachers and students miss many opportunities to make connections to real-world, understandable mathematics. The function concept unifies later study in algebra and the study of change in calculus; introducing functions earlier aids students' understanding of mathematics significantly.

Except for the Qualitative Grapher, each interactive has an associated User's Guide, "Warm-Up," and Sample Activity. The Warm Ups are simple activities that demonstrate the functions that are possible with the interactive. The Sample Activity is a challenge designed for teacher use in the online course and for their students' use. Most interactives also have FAQ's, a list of Frequently Asked Questions and their answers.

To make the interactives more useful in teaching, the McKenzie Group assembled a group of experienced mathematics teachers to develop additional classroom activities for use with the interactives. These activities are available in print form. Using a platform developed by another project at the Concord Consortium (Pallant & Tinker, 2004; Tinker, Berenfeld, & Tinker, 2000), we developed the

[4] At http://seeingmath.concord.org/sms_interactives.html

technology for integrating the McKenzie activities with the interactives so they can be delivered on-line and integrated with student assessment.[5]

VideoPaper Builder

The Seeing Math Elementary (SME) courses relied on professional videotapes of teachers to create the twelve case studies that highlighted each course. We discovered that teachers wanted not only to watch other teachers on tape, they also wanted to develop and discuss their own videos. The project created technology, VideoPaper Builder (VPB), to support communities of practice that reflect on their teaching and build their own cases in digital and interactive format.

VideoPapers are more than simply a video; they combine subtitles (because it is often difficult to follow student conversations), with text and multimedia artifacts such as student work, additional video, outlines, and standards. The multimedia paper is usually structured around the text, which contains questions and concepts that are illustrated by video segments and the other artifacts. It is, therefore, necessary to link all these elements together. In early versions of VPB, everything had to be planned beforehand and all VPB did was to link everything together. In the final version, the paper can be developed organically within VPB.

The VPB idea was piloted in Hudson, MA, where SME participants and their math coordinator shared classroom episodes. Teachers were videotaped; they screened the videos to select episodes for a case study focused on calculating the area of obtuse triangles. Teachers reviewed relevant literature, transcribed the dialog in the selected episodes, and wrote a draft paper. But it was the Concord Consortium's research group that transformed these separate pieces into a VideoPaper. The technology to support VideoPaper production was still unsophisticated.

A second version of the software was prepared and the same group from Hudson was invited to create a new VideoPaper. "Building the VideoPaper allowed us to highlight the importance of giving students time to share their mathematical thinking," said the math coordinator. However, the technology was not fully refined, and one of the teachers commented,

> *When embarking on this paper I was excited to learn about the technology and to be involved with it more. As I moved through the paper I felt overwhelmed with the technology. The programs worked and had good directions. I knew that there was support available, but even with all of that, there was just more technology than I could keep up with.*

The project created a third version of VideoPaper Builder to respond to comments such as these. This version preserved the features that allowed users to synchronize video, text and images, while increasing the application's power to create adjustable user interfaces, to handle different types of video and graphic formats, to write and edit basic HTML pages, to add captions to video segments, and to create a printable version of the hypertext with the corresponding index and references. VideoPaper Builder version 3 (VPB3) was released in October 2005. VPB3 is easy-to-use software for creating multimedia case studies. It is open source, freely available on the Seeing Math website[6], and runs on MacOSX or Windows operating systems, with interfaces and a user manual in English and Spanish. The Videopapers it creates can be viewed with any Java-enabled browser.

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[5] Pilot versions are available at <http://www.concord.org/resources/jnlp/seeing-math-applets.jnlp>

[6] Available at http://vpb.concord.org

Future educators are already benefiting from this tool. Daniel Cogan-Drew of Tufts University states that "VideoPapers have become an integral part of the pre-service teacher portfolio in the Teacher Education Program at Tufts." Working in pairs, pre-service teachers have used VPB as a means by which to develop initial research questions into their emerging classroom practice. VPB has allowed us to reflect upon and share our classroom teaching."[7]

Technology Explorations

The technology team explored the application of several technologies that hold great promise for future professional development efforts.

Highwired

Early in the project, we were approached by Highwired, an educational website that had invested over $40M into serving schools, but had become bankrupt. They offered to donate their hardware and contact lists, if we would keep their website alive. Because the original proposal envisioned a free, less functional version of Seeing Math existing in parallel with a commercial, full-featured version, Highwired seemed to offer an inexpensive vehicle for the free version. We ran the site for a period of time and analyzed the technology. However, our commercial partners were nervous about competing with a free service and we were unable to clearly define how the two would be different. Since we had no prospect of an income stream that would allow post-grant continuation, we abandoned the idea of two parallel services and with that decision, Highwired had no future, so we closed down the site.

UDL Interactive Pilot

Universal Design for Learning, or UDL, has been used extensively in reading, but not in mathematics (Rose & Meyer, 2002, 2006). The project explored how difficult it would be to take one the interactives and create a suite of different representations using the principles of UDL. This interactive could be deployed within both a web forum and as part of a downloadable application that allows teachers to create activities for their students. In addition teachers could create activities for their students and student use of the interactive could be logged and reports generated for the student, teacher, administrator, and parent.

These components in such an interactive should be able to:

summarize themselves, introspect into content, describe elements in detail
set presentation attributes: bright, subdued
output using graphics, text, speech
accept input from the keyboard, mouse, or voice.

Some of these elements rely on services provided by the operating system, such as speech output and input. The other functions need to be developed as capacities of the component itself (or as part of services our framework supplies to components). To discover how time-consuming and useful it might be, we built a prototype graph vocalizer that could describe the graph produced by the Qualitative Grapher. As a prototype, it was functional but needed considerable additional work. These

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[7] http://ase.tufts.edu/education/projects/projectVideopaper.asp

explorations lead directly to a proposal to the National Science Foundation, which was funded to develop UDL materials for use in mathematics and science.[8]

Smart Graphs

We explored a companion to the UDL designs, a Smart Graph. The idea is that the graphing software should have knowledge of the semantics of graphs. It should be able to identify for any arbitrary graph minima, maxima, recognize noise, find regions of greatest change, and know about other features typically recognized by experienced graph users. With this information, the software can either point out the salient features to the user, or generate interactive challenges that the user must solve. The software can highlight the region of a graph that represents these features. The software should also include a text output that would describe the graphs features. This could be linked to a vocalizer that speaks the description. This would be a powerful way to learn about graph interpretation and would represent a UDL tool.

During the project, we experimented with a Smart Graph prototype built from the Qualitative Grapher. As the cursor moved across the x-axis, the software explained the features of the graph at that value of x. This was most impressive, but had one major fault—when the features were dense, the speaker could not keep up with the cursor. A more interactive functionality is required.

Video Object Tracker VOT

A powerful way to introduce real graphs and real data into algebra courses is to use the ultrasonic motion detector that is now available from many vendors (Mokros & Tinker, 1987; Thornton & Sokoloff, 1990). Unfortunately, this device is expensive, so its use in mathematics is limited. The Seeing Math technology team explored the use of inexpensive USB video cameras as an alternative means of collecting data about object motion. Using tracking software, a $35 USB camera should be able to collect data of simple motion, for example: throwing a ball up in the air and catching it, rolling a ball down a ramp, or rolling a ball at different velocities off a table. Initial experimentation showed that this was possible, but that surprisingly sophisticated software was required to track a colored ball as its illumination changed. This idea was abandoned when course designers indicated that data collection was not going to be addressed in any of the courses.

Data Fitter

The Data Fitter was an interactive that was only partly developed when it became clear that no course was going to be developed to use it. The software allows students to explore best fit lines for an arbitrary set of data. There are three main objects.

- A set of data.
- A linear function that is used by the student to estimate a best fit.
- The measure of fit between the data and the line.

The data are represented in two components. The data points are displayed in the main graph area of the interactive. The x and y values of each point are also displayed in a data table. The user can change the data from each representation. The current fit (i.e., the error between the line and the data points) is shown by a "gauge," which is a visual qualitative component. This value is also showed as a numeric value below the gauge.

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[8] See http://www.concord.org/publications/newsletter/2005-spring/universal.html

SEEING MATH DISSEMINATION

Because the Concord Consortium (CC) is a nonprofit research and development group, we wanted to create and study the impact of a collection of highly innovative materials, but we realized that CC was not the right organization to scale the project up to reach the large numbers of potential course participants. For this, CC partnered with Teachscape, a startup for-profit company with significant funding for providing teacher professional development based on interactive video case studies across all subjects and grades. Teachscape was closely associated with Roy Pea, a colleague then heading SRI International's Center for Technology in Learning who provided close ties to the research community and the developments that come from that community. These partnerships have proven to be successful and enduring. The projects established a variety of strategies to ensure that the collaboration was productive. In addition to joint full-staff meetings, individual staff visited the other project when specific activities warranted personal contact to move them forward or to break a logjam. We instituted a series of weekly conference calls to maintain communications and collegiality. In addition, each team designated a project liaison (Galvis & McIntyre, 2006).

Partnership with Teachscape

Teachscape was a commercial start-up when the Seeing Math grant was awarded and grew to become profitable during the Seeing Math Project's fifth year. Teachscape contributed to the project in a number of ways, including having a model for online video case studies that Seeing Math initially used as a starting point from which to develop course content. Teachscape also provided substantial in-kind support for video case development by contributing video, editorial, and technology production services.

Previous Teachscape video cases were developed in content areas other than math (science and literacy) and our team felt that math content required different teaching strategies and, therefore, some modification to Teachscape's approach. Teachscape was open-minded and flexible; they allowed us to modify the course structure within the parameters of their platform. As a result, our video case approach slowly evolved as the Seeing Math staff garnered feedback from school district pilot sites and we incorporated that feedback in the development of both the course material and Teachscape's delivery platform.

The Seeing Math elementary cases aimed for depth in particular skill and content areas rather than breadth in topic areas. However, this presented a marketing problem for Teachscape. Their catalog of offerings in other content areas provided a substantial list of professional development titles, while the number of math-based professional development video cases was smaller.

Teachscape marketing reports focused on the problem of getting school district decision-makers to consider the Seeing Math materials because there were so few courses. When school math content staff looked at the math cases, they were consistently impressed with the extraordinary depth of the cases. It was clear that teachers could participate in only two cases a year. Therefore, a catalog of six courses could provide years of elementary math professional development.

The collaboration with Teachscape brought a new set of considerations for us. We thought that our experience with marketing a new product was unique to working with a start-up and would have been different had we partnered with an established for-profit. We found from comparing notes with other organizations and companies in the early 2000s, that even established businesses in the academic marketplace introduced new products and later had to change their focus and strategies to reflect changes in the marketplace.

As Teachscape changed focus in response to market pressures, we adjusted to their changes. They currently have more outside developers, as a direct result of the experience with the Seeing Math project. From this experience we have learned that for any partnership to work, ongoing communication is important; and in the situation where one party is the developer and the other is the marketer, it is even more important for the two parties to communicate. In this project, both for Seeing Math Elementary and Seeing Math Secondary, a tension existed between two initiatives – content and marketing – as to which should lead the design effort.

In the case of Teachscape, the content team tended to lead the design effort until there was sufficient feedback to determine that some design decisions were adversely affecting the ability to market or distribute the product. With evidence that the materials were not getting the distribution we had hoped, and with specific recommendations from the marketing staff at Teachscape, the content developers made design changes to ensure that the video case studies were used in the field.

Partnership with PBS TeacherLine

When we began our collaboration with PBS TeacherLine, both projects were mature. The projects each had strengths that complemented the other. The Concord Consortium had experience in online course development. Concord also had done a great deal of thinking about the pedagogical approach to online professional development and was recognized for its role in the creation of the Virtual High School.[9] In addition, PBS TeacherLine staff was impressed with the content of the Seeing Math Elementary materials that Teachscape was distributing. PBS TeacherLine had existing national distribution channels and marketing experience. They had already been providing online professional development to a significantly large number of teachers, and had a following of brand-loyal teachers who were interested in further online professional development opportunities.

Both projects had developed materials for elementary-level professional development. At the time they were investigating the collaboration, neither project had developed materials for the secondary level. A partnership that focused on creating and disseminating a set of secondary-level materials would help both projects meet their objectives, and would allow them to concentrate on their strengths. Moreover, it had the potential to allow both projects to learn from each other.

Dissemination

Concerned about project and product marketing and sustainability post funding, our emphasis shifted to dissemination in the final year of the project. We actively spread the word about the project, particularly in publications and websites. Additionally, we hoped to show an increase in numbers and the development of plans to ensure that the materials had a life beyond the funding cycle of the project.

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[9] See http://www.govhs.org

PBS TeacherLine found, through experimentation – in part with Seeing Math Secondary professional development courses – that the market was not interested in long courses. The optimum seems to be in the six-to-eight week range. This has been further confirmed by other online professional development providers at the Usable Knowledge conference on Online Teacher Professional Development sponsored by the Graduate School of Education at Harvard University.[10]

The price point seems less clear. There was a wide range in tuition costs for online professional development courses reported at Harvard's Usable Knowledge conference. The differences in price reflected different approaches by the offering institutions and by the audiences the courses were trying to serve. The PBS TeacherLine experience was that a unit price of about $200 (for a six-week course) was optimum for their audience. Teachscape uses a different model, which emphasizes working with a school building or district and involves providing a broader professional development approach. PBS TeacherLine was funded in 2005 for another five years in a new Ready to Teach project that will investigate a variety of approaches to delivery of their online professional development to have greater impact.

PBS TeacherLine continues to offer Seeing Math Secondary courses as part of a non-exclusive license agreement. In addition, the Concord Consortium signed a non-exclusive license agreement with Teachscape to offer our Seeing Math Secondary courses in Teachscape's course delivery platform.

A number of additional dissemination efforts were made, particularly after the fourth year of the project. These included:

The Seeing Math website: A sample of the Broken Calculator case study was posted on the project website for verified educators to see. The website itself was expanded and redesigned to include the Java interactives, and to distinguish between the secondary and the elementary school parts of the project. The site was polished, professional and marketable so it can be used as an advertising tool to draw visitors to Teachscape and PBS TeacherLine. This advertising goal was advanced through the use of a "Google Grant" which awarded us with free ad space on queries on Google such as "mathematics" or "math professional development."

CDs: The project developed compact disks containing the interactives, videos, and guides. These were distributed to teachers in a dual disc case, with one for teachers and one for students. This saved the teacher from having to download large video and document files, and allowed students to run the interactive software in computer labs without network connections.

NCTM On Math. Four articles on Seeing Math courses were published in the peer-reviewed online journal On Math, published by NCTM.

@Concord. The free newsletter *@Concord* is published twice yearly by the Concord Consortium and distribute free to 10,000 educators. Almost every issue contained one article on Seeing Math, and the spring 2006 issue featured Seeing Math, containing seven articles and a CD. All issues are available online and the CD can be downloaded from the newsletter site.

The In-Depth Study Booklet. This publication presents two in-depth studies conducted by the Seeing Math's research group (Studies #2 and #6 described in the next section). The booklet includes short versions of the in-depth studies. The attached CD-ROM gives access to com-

[10] See http://www.gse.harvard.edu/~uk/otpd/

plete digital versions of the studies, one of them in the format of a Videopaper, as well as to the VPB3 tool and manual (Galvis, 2006).

Journal of Science Education and Technology. A article about findings in online discussions at the Seeing Math Elementary project was published in 2004 by this international peer-reviewed Springer's journal (Nemirovsky & Galvis, 2004). Another article was accepted in 2006 and will be published in a forthcoming issue.

La Educ@ción. One article about findings in the pilot implementation of Seeing Math Elementary courses was published in 2004 by this international peer-reviewed online journal from the Organization of the American States (Galvis, 2004).

Converge Magazine. One invited article was prepared for the Business and Education section. It will be published in a forthcoming issue (Galvis & McIntyre, 2006).

Additional information about the publications and presentations can be found in the Appendix.

Evaluation and Research

Evaluation by Experts

Seeing Math included an evaluation of course materials by math experts as an integral part of the course development cycle. Math education specialists were consulted to review drafts of each course to ensure that they were pedagogically well designed and that the content was aligned with the math standards of the target grade level. The production team also assessed the quality of each set of course materials, testing them with representative teachers from the same grade level as the target population. These teachers reviewed the materials, giving us a real-world perspective. Their feedback informed the revisions of our content, activities, and course structure.

Evaluation through Field Tests

When course materials were complete and mounted at Teachscape (for Seeing Math Elementary courses) or at PBS TeacherLine (for Seeing Math Secondary courses), selected course materials were field tested with teachers from the target populations under actual classroom situations. The external evaluator, Edcentric, conducted surveys and visited sites to observe pilot implementations and formulate suggestions for improving the course materials, and/or their implementation. All Seeing Math Elementary and Seeing Math Secondary course materials were formatively evaluated; the feedback informed revisions of course materials, implementation strategies, or both.

The Seeing Math Elementary (SME) project recruited school districts that had agreed to participate in the project at the time the proposal was submitted. Because the funding and scope of the project was reduced from the original proposal, only four of the twenty initial school districts participated in the actual implementation and field-testing. SME implementation was driven at each school district by its own teacher professional development strategy and by the status of the SME course production. At the end of the first year of course production there were four SME courses ready to implement, and at the end of the third year a set of nine courses was available.

Through the implementation of SME courses at each of the four participating school districts (Rapid City, SD; Washington, DC; Hudson, MA; and the Windham Central Supervisory Union, VT), the external evaluator collected information concerning two of the SME cases that belong to the Number and Operations NCTM strand (courses titled Division with Remainders and Fractions) and produced the corresponding course evaluation results. These reports were complemented with yearly meetings with school district coordinators, which helped guide decision-making related to SME courses and their corresponding facilitator guides.

At the end of the second year, one school district, Rapid City, SD, had integrated SME courses into its math teacher professional development strategy and three decided to drop the project, for district-related reasons. The SME project recruited six new school districts during the third year (Champaign, IL; Dover, MA; Swampscott, MA; Mohonasen, NY; Minneapolis, MN; Barre Supervisory Union, VT) in order to continue the research. None of the initial four school districts participated in the research agenda, since three of them dropped the project and one had other math TPD initiatives running in parallel, which would have resulted in contaminated data. Only four of the six newly recruited school districts took part in the research. Edcentric collected formative evaluation data from the SME courses that were used in the math teacher professional development and research effort at the new SME school districts. Most of them used the Number and Operations NCTM strand; the remainder used the Pre-algebra NCTM strand.

The response was uniformly positive. "Formative evaluation findings were very positive, courses met or exceeded participants' expectations and course elements were considered important by the majority of the respondents" (Edcentric, 2005).

Seeing Math Secondary (SMS) course materials were ready to test at the end of the third project year. PBS TeacherLine recruited algebra teachers to field test SMS course materials and offered the first algebra course, a series addressing linear functions, transformations of linear functions, and linear equations. Edcentric formatively evaluated the materials of this course. The results were used to revise course format and implementation strategy. A second group of teachers was recruited by PBS TeacherLine, which implemented the revised version of the linear algebra course during the fall of 2004. Edcentric collected the corresponding formative evaluation information. The results did not suggest the need to revise the course materials. A third group of teachers was recruited, the course was offered by PSB TeacherLine, and Edcentric also collected formative evaluation data to confirm the quality of the materials. "These findings indicate that the Ready to Teach Algebra: Linear Family course can be an effective teacher professional development experience for experienced middle and high school mathematics teachers in helping them to explore algebra instruction for and with their students" (Edcentric & Hezzel Associates, 2005).

The Seeing Math project created a large number of elementary and secondary-level courses, more than were used for research purposes. As a consequence, in the fourth year of the project, CC recruited math educators who reflected the target population of the untested SME courses and PBS TeacherLine recruited middle and high school algebra teachers. Edcentric conducted pilot tests on these courses, using small groups or individuals who studied the materials and reported results. Teacher feedback about the materials was very positive.

Quantitative Research Studies

The Seeing Math quantitative research agenda was proposed by the Concord Consortium and approved by the Department of Education during the third year of the Seeing Math project. Research was then implemented during the remaining years of the project in collaboration with Teachscape and participating school districts using Seeing Math Elementary course materials. PBS TeacherLine recruited research teachers for Seeing Math Secondary courses. Edcentric and Hezel Associates served as external evaluators for both sets of Seeing Math materials. The complete reports are available on the Seeing Math website under "Resources."[11]

In the spring of 2004, both the Seeing Math Elementary and Seeing Math Secondary projects tried to recruit comparable treatment and control/delayed treatment teachers. It was not always possible, however, to recruit control teachers and not all the participating teachers collected the required information. Cohort 3 teachers (treatment and comparable control teachers) were targeted for recruitment at the beginning of the 2004-05 school year. Comparability of teachers was confirmed by teachers' demographic data analysis conducted by Edcentric.

The Northwest Evaluation Association (NWEA) created student tests for both the elementary and secondary sets of courses. NWEA is a nonprofit that specializes in the measurement of student math knowledge. Student tests—20-item multiple choice paper-and-pencil tests—measured math knowledge related to the NCTM standards that were directly related to the Seeing Math courses taken by participating teachers.

The Concord Consortium research group, led by an experienced math education specialist, developed the teacher knowledge tests. These assessments were field tested and adjusted before they were administered. Teacher tests were paper-and-pencil open-ended instruments, designed to measure teachers' math content and pedagogy knowledge, taken before and after participation in a Seeing Math course.

Cohort 1 teachers were invited to participate in Seeing Math courses during spring 2004 and to continue applying course ideas when teaching their students during that semester and the following school year. Cohort 2 and Cohort 3 teachers were invited to participate in Seeing Math courses during the 2004-05 school year and to apply this knowledge in teaching their students. Student and teacher pre- and post-tests for content knowledge were administered to participating teachers and their students, as follows: for Cohort 1 and Cohort 2, participants applied student tests both during spring 2004, and at the beginning and end of the school year 2004-05; Cohort 3 participants administered student tests at the beginning and at the end of the 2004-05 school year.

The SME research (Edcentric, 2005, pp. iv and v) indicated that:

> Treatment teachers performed significantly better in the content areas of Modeling/Formulating, Transforming/Manipulating, Inferring/Drawing conclusions, and Communicating than did the control teachers. Additionally, they performed significantly better than the control teachers in Pedagogy ratings. Significant differences between gains for treatment and control teachers offer evidence that participation in Seeing Math can provide teachers with relevant new learnings in a number of different content and pedagogical areas.
>
> Student findings were more difficult to interpret. Students in one district following the pre-algebra strand had significant gains on total score and both sub-scores from pre- to post-tests. However, this group did not have a control group, so no gain score comparisons could be made across treatment and control groups. Three districts used Seeing Math courses in the Number and Operations strand. In one district, there were no significant student gains for either year of the study, and no differences between treatment and control groups. In a second district, treatment students made significant gains over the second year of the study,

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[11] At http://seeingmath.concord.org/resources.html

and the treatment students had significantly higher gains than did the control students. In the third district, treatment students had significant gains in both years in some of the test scores; however, both mean scores and gains for the control students exceeded those for the treatment students for all scores.

The research on the secondary courses (Edcentric & Hezel Associates, 2005) found that:

> As a result of taking the Ready to Teach Algebra course modules, a mathematically well-prepared sample of teachers was found to learn primarily in pedagogy as opposed to specific content areas. Subsequent to taking the course, a separate and small cohort of teachers—perhaps biased by attrition—continued to learn in pedagogy and in some content areas, possibly using RTT materials and resources. Students of treatment teachers did no better overall than students of comparison teachers, but lost less in topical areas not covered by their teachers' course learning. In addition, the mathematics background of teachers had a complicated influence on these relationships.

Qualitative Research Studies

The Seeing Math project conducted a set of qualitative studies, seeking answers to issues that were important throughout the life of the project. Here, we summarize findings from the qualitative studies.

Study #1. Action Research

During the spring 2003 pilot study, it was important to collect empirical evidence related to student achievement that might be attributed to the Seeing Math professional development model. We wanted to see if we could quantify any student achievement gains related to teachers' changed classroom practices resulting from their involvement in the Seeing Math Project. An open-ended teacher action research component was developed and teachers participating in Seeing Math were invited to develop a classroom action research project they could conduct with their students.

Both the pre-test and post-test student assessments and a wealth of anecdotal evidence indicated that the Seeing Math teacher professional development model can offer almost immediate results in the classroom for participating teachers and their students. When teachers applied questioning strategies and an inquiry-based approach to teaching mathematics in their classes, students increased their problem-solving skills and math understanding (Gadzuk, 2003).

Study #2. Critical Success Factors

Lessons learned in the Seeing Math project and earlier work were distilled into a document that describes "Critical Success Factors" (Galvis, 2004). These factors describe the conditions are necessary for success when implementing case-based teacher multimedia professional development. Our findings grow out of data collected during the first pilot implementation with four school districts. The districts achieved different levels of success, mediated by several variables. The research focused on these variables, teasing out and identifying those that may make a difference in the success or failure of an innovation at different stages of implementation.[12]

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[12] Full text version available at http://seeingmath.concord.org/images/040413CSFarticle.pdf

Study #3. Video-Based Online Discussions

The use of interactive video cases for teacher professional development is an emergent medium inspired by case study methods used extensively in law, business, and medicine, and by the advent of technology available to support online discussions.

The project identified web-based "grounded" discussions as critical to successful case-based courses. A grounded discussion is on in which the participants base their contributions on specific events portrayed in the case, as well as the role facilitators play in these online interactions. To make this point we analyzed the online exchange of messages in one school district that participated in Seeing Math (Nemirovsky & Galvis, 2004).

Study #4. Facilitated Discussions

The examination of the discussion forums in Seeing Math Elementary and Seeing Math Secondary, despite the differences in platform and course structure, pointed out the following commonalities observed in the online discussions in the pilot test (Gadzuk, 2005):

The value of templates or starter threads for facilitators to prompt discourse within the discussion boards. These templates helped maintain "quality control" and some level of consistency across the class sections, regardless of the facilitators' expertise, degree of involvement, or time spent actively facilitating. Despite these facilitation aids, however, there was considerable variation among the facilitators in their guidance and expectations for participating teachers.

The salience of facilitator feedback. Even though we, as evaluators, did not have access to the "personal feedback" areas of the courses, individual emails, or telephone records, we believe that facilitators who provided extensive online feedback and who devoted more time to their work as facilitators were also providing additional extensive feedback through private venues and face-to-face meetings. As demonstrated by the example of understanding the distinction between functions and equations – one of the primary math concepts of the algebra course - teachers in groups with more facilitator feedback were more likely to express the belief that they understood this distinction and would be able to apply it in their classrooms.

The logistical problems raised by a course structure with an explicit schedule requiring a prescribed number of teacher posts on separate days. The working reality of busy teachers is that most are able to put the majority of their time into the course only on weekends. This resulted in a "traffic jam" in the discussions on Sunday and Monday nights (the end of the course week) rather than a smooth flow of deep, reflective thought spread throughout the week. As a result, much of the "deep" thinking presented in the posts could be seen as reflective monologue rather than reflective dialog. Participants had the opportunity to reflect on their own mathematical thinking and pedagogical concerns and present them to their discussion group, but there was not the opportunity to build on each other's responses as deeply as had been hoped. The time frame frequently forced these insights to develop in parallel among the participants, rather than allowing the insights to build cumulatively or collectively based on group reflections on each other's posts.

Study #5. Teaching Practice

We made an in-depth study of teachers participating in the Broken Calculator Seeing Math course. Through classroom observations, videotapes, interviews and course discussion postings, we examined the influences of participation in the course on math teaching practices and student learning (Metcalf, Nemirovsky, Griffin-Zuniga, & Galvis, 2005).

Study #6. Impact of Participation in SME

We created a VideoPaper (Nemirovsky, Metcalf, Galvis, & Griffin-Zuniga, 2006) from the class of Anne, a third grade teacher in a suburban Massachusetts public school. The first section provides background information about the study and the Seeing Math project in which Anne and her school participated. The following three sections present video segments and analysis. Each part focuses on a different theme: the role of stories in problem solving, the interplay between conventions and strategies, and the aspects involved in judging a solution beyond being right or wrong.

Appendix: Seeing Math Products

Seeing Math Elementary Courses

Effective Questioning in the Mathematics Classroom

In Effective Questioning in the Mathematics Classroom we examine how a teacher's questioning practices can elicit student thinking to advance student understanding. Because this course connects particular forms of questions to specific purposes, and invites teachers to reflect on their own question styles, the strategies in this course can be applied broadly across mathematical content and grade levels, unlike most content-focused professional development efforts.

Foundations of Effective Mathematics Teaching

This course highlights the rationale and pedagogy underlying three key aspects of effective teaching: teacher content knowledge, listening to and interpreting student thinking, and formative assessment that guides future instruction.

Formative Assessment in the Mathematics Classroom

While much attention has been given to summative assessment strategies, the awareness and practice of formative assessment techniques remains largely underemphasized. This course presents a wide range of strategies and rationales for using formative assessment to evaluate student understanding in the mathematics classroom and provides many opportunities for teachers to observe other teachers and reflect on their own practices.

Number & Operations: Division with Remainders

In Number and Operations: Division with Remainders, we visit the classroom of students learning about the nature of division by creating and solving their own story problems. This teaching focuses on laying a conceptual foundation for division before presenting a formal algorithm.

Number & Operations: The Magnitude of Fractions

In Numbers & Operations: The Magnitude of Fractions, we expand the concept of fractions. Students already understand fractions as parts of a whole. By comparing fractions, students come to understand them as something more—numbers that have magnitude. This is the central goal of the lesson.

Number & Operations: Broken Calculator

In Number & Operations: Broken Calculator, students develop computational fluency by solving problems with a "broken" calculator. For example, a baker packs 315 buns into boxes of 8. How many boxes will the baker need? The division key is broken! Students must find ways to solve the problem, evaluate the solutions, and explain their reasoning.

Geometry: 2D and 3D Figures

In Geometry: 2D and 3D Figures, students explore the terminology and properties of three-dimensional solids. They visualize solid figures, learn the names of their parts, and build and draw figures in two and three dimensions.

Geometry: Calculating Area of a Triangle

In Geometry: Calculating the Area of a Triangle, a group of fifth grade students builds a foundation for understanding the area of a triangle and finds methods to calculate it before learning the standard formula.

Data Analysis and Probability: Using Data to Make Predictions

In Data Analysis and Probability: Using Data to Make Predictions, we present ways to support NCTM standards for grades 3-5 that invite students to collect, analyze, and make predictions from data. The teaching case presents two lessons taught to a fifth grade class. The first lesson explores mathematical fairness. The second lesson highlights the relationship between sample size and accuracy of predictions about a whole population.

Data Analysis and Probability: Measures of Center

In Data Analysis and Probability: Measures of Center, we visit a classroom of students looking at ways of measuring and describing data and data sets. A note of interest—all the students in this class are English Language Learners (ELLs) or speakers of native languages other than English.

Pre-Algebra: Pan Balance Equations

In Pre-Algebra: Pan Balance Equations, a fifth grade class manipulates concrete representations of equivalence, using pan balances, to understand the nature of equations and operations on equations. Students investigate a model for solving equations using one and two variables.

Pre-Algebra: Patterns and Functions

In Pre-Algebra: Patterns and Functions, a fourth grade class explores the concept of function by analyzing a sequence of shapes. Using tiles, pictures, T-charts, and linear graphs students try to figure out a growth rule, or function, that allows them to make the next member of the sequence and predict how many tiles it has.

Seeing Math Secondary Courses

Proportional Reasoning

Proportional Reasoning prepares the transition from a primary focus on arithmetic and skills with algorithms (typical of elementary and middle school) to a focus on algebra, where students use multiplicative, as well as additive, thinking. Goals:

Explore the relationship between proportional reasoning and algebraic thinking.

Understand that students who appear to reason proportionally may in fact be following a procedure without understanding it.

Understand methods for developing students' proportional reasoning.

Linear Functions

Linear Functions introduces algebra through the mathematically cohesive concept of functions and grounds it by modeling real-life situations. Goals:

Interpret the meaning and characteristics of linear functions in the context of real-world situations

Identify ways in which multiple representations can express and enrich mathematical concepts

Use piecewise functions to accentuate characteristics of linearity in the context of real-world modeling situations

Transformations of Linear Functions

Participants observe relationships between graphic and symbolic forms of a function. They explore how changes to the graphic representation of a function alter its symbolic representation, and vice versa. Goals:

Observe graphic and symbolic transformations of linear functions

Categorize, use, and represent families of linear functions in multiple formats

Interpret the concept of slope in different contexts

Linear Equations

Most algebra curricula introduce linear equations before linear functions. In Linear Equations, functions are discussed first. Learners conceive and manipulate equations as particular values of a general pattern—functions. Goals:

Understand the rationale behind the rules of symbol manipulation that maintain an equality or corresponding inequality

Deepen the distinction between equivalence of function and equality of value

Gain facility in moving easily between symbolic and graphic techniques for solving equations and inequalities, whether presented in symbolic or story (text) form

Systems of Linear Equations

Participants define a linear function in two variables that is expressed in the form Ax+By = C. They then compare the graphs of two or more functions of this form, and explore graphically how some, but not all, algebraic operations leave the solution set of the system unchanged. Participants also look at traditional methods of solving systems of linear equations by substitution and by combination. Goals:

Use the function approach to understand how graphic and algebraic methods relate when solving systems of equations

Explore the effects of algebraic operations on solution sets, using either individual linear functions or pairs

Use graphic representations to model real world applications of systems of linear functions

Quadratic Functions

Participants in Quadratic Functions use models and problem solving to examine how the general nature of quadratic functions informs the particular instances described by quadratic equations. Participants also use multiple representations—tables, graphs, and equations—as powerful tools to describe physical situations. Goals:

Search for patterns and use quadratic functions to model physical situations

Interpret the meaning and characteristics of quadratic functions as they appear in different representations

Link a personal understanding of quadratic functions to your curriculum and to students' understanding

Transformations of Quadratic Functions

Using interactives, participants observe how changing symbolic expressions alters their graphic representation, and vice versa. By working with families of quadratic functions, they deepen their understanding of the role of each symbolic form in gleaning information about a function. Goals:

Represent, categorize and use families of quadratic functions in multiple formats

Understand how each of the three major symbolic forms — polynomial, root (product), and vertex — serve a different purpose and how each relates to graphic transformations

Exploit function notation as a useful tool to visualize the relationships among related quadratic functions

Quadratic Equations

In Quadratic Equations, we make the relationship between quadratic functions and quadratic equations explicit. Because textbooks and tests devote a great deal of time to the skills of factoring and finding roots, participants also use graphical means, as well as successive approximations in tabular form, to reach the same goal. Goals:

Identify the three symbolic forms for quadratic equations—polynomial, product, and vertex—and clarify how each form's role informs not only its graphic representation, but how to find the root of its function

Delineate techniques for solving quadratic equations by comparing quadratic functions, both graphically and symbolically

Solve quadratic inequalities, in both graphic and symbolic forms, as a natural extension of comparing functions

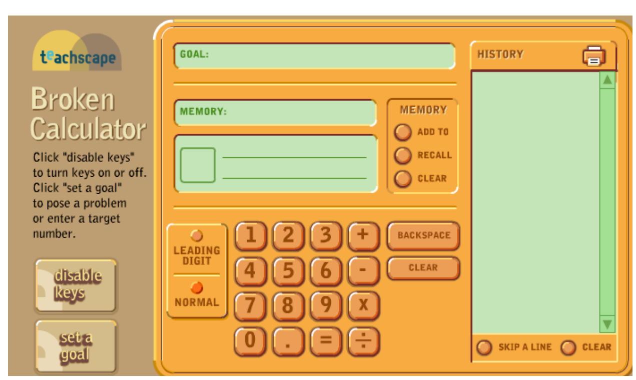
Data Analysis

Data Analysis addresses approaches to measures of center (mean, median, mode) and how they describe a data set. Students often use the algorithms associated with mean and median without fully understanding the ideas. When reasoning about data, they often attend to single cases and fail to observe characteristics of the aggregate, such as measures of center, as describing or representing the data set. Goals:

Listen for and support deeper understanding of measures of center and descriptors of the aggregate

Review a range of graphic representations and their applications

Explore reasoning about data aggregates through indices of data, different graphic representations, and an interactive tool



Seeing Math Interactives

Broken Calculator

The course SME course "Number & Operations: Broken Calculator" examines how broken calculator problems can help students develop computational fluency. Faced with the challenge of solving problems without using certain disabled keys on their calculators, students must think creatively and use a variety of skills. They use number facts and estimation; they break numbers apart and recombine them; they use alternative operations to work around "broken" keys. They show each other how to use "friendly" numbers to approximate answers, and then figure out how to compensate to get the exact solution. Some problems require that students break multiplication problems into component parts and understand in detail how multi-digit multiplication works.

The interactive software that enables participants to:

Disable certain number or function keys on the calculator
Set "goals"—either target numbers to reach or problems to solve
Maintain a history of steps taken to solve problems.

Qualitative Grapher

This very simple applet highlights the meaning of a function, and allows the user to see how a graph can be seen as something changing over time. The tool links a motion model to the graph that a user creates.

Piecewise Linear Grapher

This applet focuses on the language of domain and range, and the ideas of continuity and discontinuity. The software links symbolic and graphic representations of each interval of a piecewise linear function.

Linear Transformer

This interactive links the meaning of each component of a linear function's symbolic expression with its symbolic and graphic representations. Interactions include translating (dragging) a line vertically or horizontally, rotating it around a fixed point, or reflecting it around the x- or y-axis.

Function Analyzer

This applet allows the user to explore the rationale behind symbolic operations used to solve a linear equation. The user can change the graphic and area models of functions as you change the value of each symbolic element.

Quadratic Transformer

This package is like the *Linear Transformer*, but works with quadratics. Each component of a quadratic function's symbolic expression is linked to symbolic and graphic representations of translating (dragging) a parabola vertically or horizontally, dilating it, or reflecting it around the x- or y-axis.

System Solver

This interactive allows the user to explore symbolic solutions of systems of linear equations and to see how symbolic operations on a system of linear equations do (or do not) change the graphic or tabular representations of the system. The software is a tool intended to illustrate the rationale behind the symbolic operations used to solve systems of linear equations, and not a way to learn what procedures to follow.

Plop It!

This simple tools shows how changing a data set affects the mean, median, and mode (created by The Shodor Education Foundation[13] and modified by The Concord Consortium).

Proportioner

This tool highlights proportion and scale by allowing the user to compare image dimensions by using one image to "paint" another.

Online Student Activities

Student activities including embedded formative assessment using the Molecular Workbench Editor for the following interactives:

- Qualitative Grapher
- Piecewise Linear Grapher
- Linear Transformer
- Function Analyzer
- Quadratic Transformer
- System Solver
- Plop It!
- Proportioner

Publications

Partnering to Offer Multimedia Case-based Math Teacher Professional Development. Converge Magazine. (Invited article, forthcoming)

What is 21st Century Secondary Math? Concepts, Not Computation. @Concord, Fall 2006.

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Transforming Linear Graphs: Linking Symbols, Forms, and Graphs, NCTM On-Math 2006, Volume 4, Number 1

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An Interactive Approach to Transforming Parabolas, NCTM On-Math 2006, Volume 4, Number 1

Solving Systems of Linear Equations: Linking Symbolic Manipulations, Graphs, and Solutions, NCTM On-Math 2006, Volume 4, Number 1

How Important is the Online Facilitator? Seeing Math offers Moderator-Lite Scalable Professional Development, @Concord Spring 2006

Teachscape and PBS TeacherLine Offer Seeing Math Courses, @Concord Spring 2006

Interaction and Interactivity, @Concord Spring 2006

Seeing Math Research: Promising Gains, @Concord Spring 2006

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[13] Used by permission. See http://www.shodor.org/

Are Online Courses Effective for Professional Development? Lessons from a Decade of Experimentation, @Concord Spring 2006

Lights, Camera, Action: Videotaping Teachers for Professional Development, @Concord Spring 2006

Improving Student Learning with Teacher Professional Development, @Concord Fall 2005

Tuesday's Lesson: What Can You Do with a Broken Calculator?, @Concord Fall 2005

Wednesday's Lesson: The Trickster Squirrel, @Concord Fall 2005

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Thursday's Lesson: Warming Up to Quadratics with the Parabola Web, @Concord Fall 2004

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Collison, G., Collison, J., Harik, F. Transforming Linear Graphs: Linking Symbols, Forms, and Graphs. ON-Math 2006 | Volume 4, Number 1

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NCTM 2005, National Council of Teachers in Education Conference

NCSM 2005, National Council of Supervisors of Mathematics

NECC 2005, National Education Computing Conference

SITE 2005, Society for Information Technology and Teacher Education

ED-MEDIA 2005, World Conference on Educational Media, Hypermedia and Telecommunications

TECH_ED 2005, Technology in Education

CCEC 2005 Congreso por la Calidad de la Educación Colombiana

INEAM summit 2005

LACDE 2005, Latin American Conference on Distance Education

ELEARN 2004, World Conference on e-Learning in Corporate, Government, Healthcare and Education

SITE 2004

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NCTM 2004 (Exhibit)

NECC 2004 (Exhibit)

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http://seeingmath.concord.org/index.html

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